Published by Institute of Physics Publishing for SISSA/ISAS
Received: *September 6, 2002*
Revised: *November 13, 2002*
Accepted: *November 18, 2002*

Gauge coupling renormalization in orbifold field theories

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Abstract: We investigate the gauge coupling renormalization in orbifold field theory preserving 4-dimensional $N = 1$ supersymmetry in the framework of 4-dimensional effective supergravity. As a concrete example, we consider the 5-dimensional Super-Yang-Mills theory on a slice of AdS_5. In our approach, one-loop gauge couplings can be determined by the loop-induced axion couplings and the tree level properties of 4-dimensional effective supergravity which are much easier to be computed.

Keywords: Renormalization Group, Field Theories in Higher Dimensions, Supersymmetric Effective Theories.

Contents

1. Introduction

Recently higher-dimensional field theories compactified on orbifold have been proposed as models providing an efficient mechanism for symmetry breaking, e.g. the supersymmetry (SUSY) breaking [1, 2] and/or the grand unified gauge symmetry breaking [3]. One can construct realistic grand unified models more efficiently in such framework [4, 5]. Higher-dimensional orbifold models may also lead to a geometric understanding of various hierarchical mass scales in particle physics [6, 7], the suppression of some Yukawa couplings [8], and the b-t mass ratio [9].

In this paper, we wish to discuss the gauge coupling renormalization in orbifold field theories preserving the 4-dimensional (4D) $N = 1$ supersymmetry in the framework of 4D effective supergravity (SUGRA). We are particularly interested in theories with large scale hierarchies, for instance a model in which the Kaluza-Klein (KK) threshold scale is significantly lower than the cutoff scale of the theory so that the KK towers are important for the gauge coupling renormalization. In such cases, it is quite convenient to consider the gauge coupling renormalization in the framework of 4D effective SUGRA since the 1-loop gauge couplings can be determined by the *loop-induced axion couplings* and the *tree level properties* of 4D effective SUGRA which are much easier to be computed. As a concrete example, we will consider 5D SUGRA-coupled super-Yang-Mills (SYM) theory on a slice of 5D Anti-de Sitter space (AdS_5) [10]–[14] with four well-seperated mass scales: the KK scale, the orbifold length, the AdS curvature, and finally the cutoff scale. However, much of the discussions here can be easily extended to generic higher dimensional orbifold field theories. Also the results for a flat supersymmetric 5D geometry can be obtained from our AdS results by taking the limit that the AdS curvature becomes zero.

The organization of this paper is as follows. In section 2, we discuss some features of the supersymmetric gauge theory on a slice of AdS_5, including the gauged $\mathrm{U}(1)_R$ symmetry and also the possible form of gauge coupling renormalization. In section 3, we derive the

4D effective SUGRA of the 5D theory on AdS_5 and match the renormalized low energy gauge couplings in 5D theory with the moduli-dependent gauge couplings in 4D effective SUGRA. With this matching, one can determine the 1-loop gauge couplings by the 1-loop induced axion couplings and the tree level properties of 4D effective SUGRA. Section 4 is the conclusion.

2. Supersymmetric gauge theory on a slice of AdS_5

In this section, we discuss some features of the SUGRA coupled SYM theory on a slice of AdS_5 which is orbifolded w.r.t. $y \to -y$ and $y \to y + 2\pi$ where y is the coordinate of the 5-th dimension. The model under consideration contains 5D vector multiplets for YM fields, 5D hypermultiplets for charged matter fields, as well as the 5D SUGRA multiplet. To be general, we also assume that there are charged brane matter fields living on 3-branes at the orbifold fixed points $y = 0$ and π.

The action of the SUGRA multiplet is given by [10, 11]

$$
\begin{aligned}
S_{\rm sugra} = -\frac{1}{2}\int d^4x dy \sqrt{-G} M_*^3 \Big\{ & \mathcal{R} + \bar{\Psi}^i_M \gamma^{MNP} D_N \Psi_{iP} + \frac{3}{2} C^{MN} C_{MN} - \\
& - \frac{1}{4\sqrt{-G}} \epsilon^{MNPQR} B_M C_{NP} C_{QR} - \frac{3}{2} k \epsilon(y) \bar{\Psi}^i_M \gamma^{MN} (\sigma_3)_{ij} \Psi^j_N - \\
& - 12k^2 + \frac{(\delta(y) - \delta(y-\pi))}{\sqrt{G_{55}}} 12k + \cdots \Big\} ,
\end{aligned}
\tag{2.1}
$$

where $\mathcal{R}$ is the 5D Ricci scalar for the metric G_{MN}, $C_{MN} = \partial_M B_N - \partial_N B_M$ is the field strength of the graviphoton B_M, Ψ^i_M $(i = 1, 2)$ are the symplectic Majorana gravitinos, M_* is the 5-dimensional Planck scale, k is the AdS curvature, and the ellipsis stands for the higher dimensional terms which are not relevant for the present discussion. Here the gravitino kink mass and the brane cosmological constants are determined by supersymmetry, and the indices i, j label the fundamental representation of the $SU(2)_R$ automorphism group of 5D SUGRA, which is raised or lowered by $\varepsilon^{ij} = \varepsilon_{ij} = (i\sigma_2)^{ij}$ in the NW-SE convention. The SUGRA multiplet has the standard Z_2 boundary condition:

$$
\begin{aligned}
e^A_M(-y) &= Z^A_B Z^N_M e^B_N(y) , \\
\Psi^i_M(-y) &= Z^N_M (\sigma_3)^i{}_j \gamma_5 \Psi^j_N(y) , \\
B_M(-y) &= -Z^N_M B_N(y) ,
\end{aligned}
\tag{2.2}
$$

where e^A_M is the 5-bein and

$$
Z^N_M = Z^B_A = \mathrm{diag}(1, 1, 1, 1, -1).
$$

The above orbifolding breaks the $N = 2$ SUSY down to $N = 1$. In this paper, we consider only the orbifolding preserving $N = 1$ SUSY, so all gravity multiplets are assumed to be periodic under $y \to y + 2\pi$. The above action for SUGRA multiplet leads to the following AdS_5 metric as a solution of the equations of motion:

$$
ds^2 = e^{-2k|y|R} g_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2 , \tag{2.3}
$$

where $g_{\mu\nu}$ denotes the massless 4D graviton and R is the orbifold radius.

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The action of vector multiplets is given by [11]

$$
\begin{aligned}
S_{\text{vector}} = -\frac{1}{2}\int d^4x dy \sqrt{-G}\Big\{ & \frac{1}{2g_{5a}^2}F^{aMN}F^a_{MN} + D_M\phi^a D^M\phi^a + \bar{\lambda}^{ia}\gamma^M D_M\lambda^a_i + \\
& + \frac{1}{4\sqrt{-G}g_{5a}^2}\epsilon^{MNPQR}B_M F^a_{NP}F^a_{QR} + \frac{1}{2}k\epsilon(y)\bar{\lambda}^{ia}(\sigma_3)_{ij}\lambda^{ja} - \\
& - 4k^2\phi^a\phi^a + \frac{\delta(y)}{\sqrt{G_{55}}}\left(\frac{1}{2g_{0a}^2}F^{a\mu\nu}F^a_{\mu\nu} + 4k\phi^a\phi^a + \cdots\right) + \\
& + \frac{\delta(y-\pi)}{\sqrt{G_{55}}}\left(\frac{1}{2g_{\pi a}^2}F^{a\mu\nu}F^a_{\mu\nu} - 4k\phi^a\phi^a + \cdots\right) + \cdots\Big\}, \qquad (2.4)
\end{aligned}
$$

where F^a_{MN} are the YM field strengths, λ^{ia} are the symplectic Majorana gauginos, ϕ^a are real scalar fields spanning a very special target manifold, and the ellipses denote the higher dimensional terms. Again the gaugino kink mass and the scalar (bulk and brane) masses are determined by supersymmetry. Here we are interested in the vector multiplets containing zero mode gauge fields, so the Z_2 boundary conditions are given by

$$
A^a_M(-y) = Z^N_M A^a_N(y)\,, \qquad \lambda^{ia}(-y) = (\sigma_3)^i{}_j\gamma_5\lambda^{ja}(y)\,, \qquad \phi^a(-y) = -\phi^a(y)\,, \qquad (2.5)
$$

where all fields in the vector multiplet are assumed to be periodic under $y \to y + 2\pi$.

The action of hypermultiplets has the form [11]

$$
\begin{aligned}
S_{\text{hyper}} = \int d^4x dy\sqrt{-G}\Big[& |D_M h^i_I|^2 + i\bar{\Psi}_I\gamma^M D_M\Psi_I + ic_I k\epsilon(y)\bar{\Psi}_I\Psi_I + \sum_I\sum_i \times \qquad (2.6)\\
& \times\left(\left(c_I^2 + \epsilon_i c_I - \frac{15}{4}\right)k^2 + k(3-2\epsilon_i c_I)\frac{(\delta(y)-\delta(y-\pi))}{\sqrt{G_{55}}}\right)|h^i_I|^2 + \cdots\Big]
\end{aligned}
$$

where h^i_I are two complex scalar fields in the I-th hypermultiplet spanning a quaternionic target manifold with the tangent space group $\mathrm{SU}(2)_R \times Sp(2n_H)$ ($I = 1, \ldots, n_H$), Ψ_I are the Dirac fermions with kink mass $c_I k\epsilon(y)$, and $\epsilon_{1,2} = \pm 1$. Under the $[\mathrm{U}(1)]^{n_H}$ subgroup of $[\mathrm{SU}(2)]^{n_H} \subset Sp(2n_H)$, the hypermultiplets transform as[1]

$$
h^i_I \to e^{i\alpha_I}h^i_I\,, \qquad \Psi_I \to e^{i\alpha_I}\Psi_I,
$$

while the $\mathrm{SU}(2)_R$ transformation can be read off from the index i. Here the hypermultiplets are allowed to have nontrivial boundary conditions for both $y \to -y$ and $y \to y + 2\pi$:

$$
\begin{aligned}
h^i_I(-y) &= \omega_I(\sigma_3)^i{}_j h^j_I(y)\,, \qquad & h^i_I(-y+\pi) &= \eta_I(\sigma_3)^i{}_j h^j_I(y+\pi)\,, \\
\Psi_I(-y) &= \omega_I\gamma_5\Psi_I(y)\,, \qquad & \Psi_I(-y+\pi) &= \eta_I\gamma_5\Psi_I(y+\pi)\,, \qquad (2.7)
\end{aligned}
$$

where $\omega_I = \pm 1$ and $\eta_I = \pm 1$. Note that $y + \pi \to -y + \pi$ corresponds to the successive transformation of $y \to -y$ and $y \to y + 2\pi$.

[1]Here we choose the scalar field fluctuations not being the coordinates of the quaternionic target manifold, but being a fundamental representation of the tanget space group $\mathrm{SU}(2)_R \times Sp(2n_H)$, which is always possible with the $4n_H$-beins on the quaternionic manifold.

To obtain supersymmetric AdS background, we need to gauge $\mathrm{U}(1)_R$ symmetry by graviphoton. It has been noted in [15] that the gauge coupling of B_M is required to be Z_2-odd when the 5D SUGRA model contains matter fields and nontrivial brane actions. Then the $\mathrm{U}(1)_R$ gauge transformation and the corresponding covariant derivative are given by

$$\begin{aligned}\Phi &\to e^{-i\epsilon(y)\Omega(x,y)g_BT_B}\Phi\,,\\ B_M &\to B_M+\partial_M\Omega\,,\\ D_M\Phi &= \partial_M\Phi+i\epsilon(y)g_BT_BB_M\Phi\,,\end{aligned}\tag{2.8}$$

where g_B is a coupling constant, T_B is the gauged $\mathrm{U}(1)_R$ generator, and Ω is a continuous gauge function obeying the orbifolding boundary condition

$$\Omega(y)=-\Omega(-y)=\Omega(y+2\pi)\,.$$

Note that $\Omega(0)=\Omega(\pi)=0$ guarantees that (i) $D_M\Phi$ has the same gauged $\mathrm{U}(1)_R$ transformation as Φ, (ii) the Chern-Simons terms in the action are invariant under the gauged $\mathrm{U}(1)_R$, (iii) there is no $\mathrm{U}(1)_R$-anomaly. It also means that bulk and brane matter fields at $y=0,\pi$ are all invariant under the gauged $\mathrm{U}(1)_R$.

In order for the $\mathrm{U}(1)_R$ covariant derivative to be consistent with the SUSY transformation, T_B is required to commute with the Z_2 transformation associated with $y\to -y$. Without loss of generality, such $\mathrm{U}(1)_R$ generator can be written as[2]

$$g_BT_B=g_RT_{3R}+g_IT_{3I}\,,\tag{2.9}$$

where T_{3R} is the U(1)-generator of $\mathrm{SU}(2)_R$ and T_{3I} is the U(1)-generator of the I-th SU(2) subgroup of $Sp(2n_H)$. It is then straightforward to find (see appendix)

$$g_R=-3k\,,\qquad g_I=c_Ik\,,\tag{2.10}$$

yielding the following form of covariant derivatives:

$$\begin{aligned}D_Mh^i_I &= \nabla_Mh^i_I-i\left(\frac{3}{2}(\sigma_3)^i{}_j-c_I\delta^i{}_j\right)k\epsilon(y)B_Mh^j_I\,,\\ D_M\Psi_I &= \nabla_M\Psi_I+ic_Ik\epsilon(y)B_M\Psi_I\,,\\ D_M\lambda^{ia} &= \nabla_M\lambda^{ia}-i\frac{3}{2}(\sigma_3)^i{}_jk\epsilon(y)B_M\lambda^{aj}\,,\end{aligned}\tag{2.11}$$

where ∇_M denotes the covariant derivative containing other gauge fields including the spin connection.

In addition to $S_{\rm sugra}$, $S_{\rm vector}$ and $S_{\rm hyper}$, there can be additional brane actions involving the brane fields as well as the bulk fields at the fixed points. Such brane actions are required to be invariant under the unbroken $N=1$ SUSY generated by the Killing spinor on AdS_5 which will be discussed in appendix. Also the gauged $\mathrm{U}(1)_R$ enforces that B_M can appear

[2] Note that T_B can always be matched with an isometry generator in the quaternionic manifold of hypermultiplet scalar fields.

in the brane actions only through $C_{\mu 5} = \partial_\mu B_5 - \partial_5 B_\mu$, not through the covariant derivative of matter field. As a result, the brane actions do not have any non-derivative coupling of B_5 in the field basis of $S_{\rm vector}$ and $S_{\rm hyper}$.

Let us now consider the mass scales involved in the model. Generically, orbifold field theories on a slice of AdS$_5$ have the KK scale, i.e. the scale where the massive KK modes start to appear and/or the KK level spacing, given by

$$M_{KK} \simeq \frac{\pi k}{e^{\pi k R} - 1}\,. \tag{2.12}$$

In the limit of large AdS curvature $\pi k R \gg 1$, the KK scale is exponentially suppressed, $M_{KK} \simeq \pi k e^{-\pi k R}$, so we have the mass scale hierarchies[3]

$$\mu \ll M_{KK} \ll 1/R \ll k \ll M_*\,, \tag{2.13}$$

where μ denotes the low energy scale for currently available experiments.[4] In the other limit with small AdS curvature $\pi k R \ll 1$, the geometry is (approximately) flat and the KK scale is given by

$$M_{KK} \simeq 1/R\,, \tag{2.14}$$

for which the scale hierarchies are given by

$$\mu \ll M_{KK} \simeq 1/R \ll M_*\,. \tag{2.15}$$

The low energy couplings of gauge field zero modes in AdS$_5$ would appear as a dimensionless function of the involved mass scales M_{KK}, R, k, M_* as well as of the bare couplings $g_{5a}^2, g_{0a}^2, g_{\pi a}^2$ [13, 14]. (Here we assume that M_* corresponds to the cutoff scale of the model.) At tree level, the 4D gauge couplings g_a^2 are simply given by

$$\left(\frac{1}{g_a^2}\right)_{\rm tree} = \frac{\pi R}{g_{5a}^2} + \frac{1}{g_{0a}^2} + \frac{1}{g_{\pi a}^2}\,.$$

At 1-loop order, there can be two types of quantum corrections: those which are power-law dependent on the involved energy scales and the others which are logarithmic in scales. As for the power-law dependent part, it is dominated by the contribution from the cutoff scale M_*, while the logarithmic part receives equally important contributions from all scales. When $\pi k R \gg 1$ so that we have the mass hierarchy (2.13) with $M_{KK} \approx \pi k e^{-\pi k R}$, writing the dimensionless 1-loop 4D gauge coupling at low energy scale $\mu \ll M_{KK}$ in terms of the involved mass scales in a manner having sensible limiting behavior at large R, we find that

[3] It is normally assumed that $k \sim M_* \sim M_{\rm Pl}$ and $kR \sim 10$ in Randall-Sundrum scenario, generating a large scale hierarchy $k/M_{KK} \sim e^{\pi k R} \sim 10^{16}$ without having any large scale ratio between the fundamental mass parameters.

[4] Our orbifold field theory has many scalar fields, e.g. scalar fields in 5D vector or hypermultiplets, which can have nonzero vacuum expectation values (VEV) $\langle\sigma\rangle$ in general. In case that $\langle\sigma\rangle \gg \mu$, we should take into account this additional mass scale in the analysis. Here we assume that there is no such mass scale, so no more mass scales other than M_{KK}, $1/R$ and k between μ and M_*.

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$g_a^2(\mu)$ can be generically written as[5]

$$\frac{1}{g_a^2(\mu)} = \left[\frac{1}{g_{5a}^2(M_*)} + \frac{\gamma_a}{8\pi^3}M_*\right]\pi R + \frac{1}{g_{0a}^2(M_*)} + \frac{1}{g_{\pi a}^2(M_*)} + \frac{C_a}{8\pi^2} + \frac{b_a'''}{8\pi^2}\ln\left(\frac{M_*}{k}\right) + \\ + \frac{b_a''}{8\pi^2}\ln(kR) + \frac{b_a'}{8\pi^2}\ln\left(\frac{1}{M_{KK}R}\right) + \frac{b_a}{8\pi^2}\ln\left(\frac{M_{KK}}{\mu}\right), \tag{2.16}$$

where C_a are some constants which do *not* depend on any of μ, M_{KK}, R and k at one-loop approximation, and b_a are the conventional one-loop beta function coefficients receiving the contribution only from the massless 4D modes at scales below μ. The other coefficients b_a', b_a'', b_a''' and γ_a receive the contribution from the KK towers. Among them, b_a', b_a'' and b_a''' can be unambiguously calculated within the orbifold field theory as they reflect the infrared property of the model below the cutoff scale M_*, while γ_a are uncalculable as they reflect the unknown UV physics around M_*. We stress that the logarithms $\ln(M_I/M_J)$ in (2.16) for $\{M_I\} = \{M_*, k, 1/R, M_{KK}\}$ can *not* be interpreted solely as the coupling running between M_I and M_J. There can be some contribution from the coupling running, particularly from the running localized at the orbifold fixed points. However they include also the finite KK threshold corrections. So (2.16) should be interpreted as one simple way to reorganize the whole quantum corrections including both the KK threshold effects and the running effects. The appearance of $\ln(k)$ and $\ln(R)$ in (2.16) may look unusual since k and R can be considered as purely geometrical objects. However k corresponds to a physical threshold scale in the sense that the physics below k has 4D CFT dual, while the physics above k is rather close to 5D physics with flat extra dimension. As for $\ln(R)$, it can appear through the wavefunction coefficients of the y-independent zero modes, i.e. the tree level $1/g_a^2$ and the Kähler metric of the zero mode of H_I (H_I^c) with $c_I \approx 1/2$ $(-1/2)$, which are proportional to R and affect the low energy gauge couplings through their logs.

In case with large scale ratios given by (2.13), the large logs of b_a, b_a' and b_a'' provide important corrections to the 4D gauge couplings. As for C_a which are independent of mass scales, they are generically of order unity, so subleading compared to the large-logs. One can also make a strong coupling assumption[16] on the uncalculable bare brane couplings

$$\frac{1}{g_{0a}^2(M_*)} \sim \frac{1}{g_{\pi a}^2(M_*)} = \mathcal{O}\left(\frac{1}{8\pi^2}\right), \tag{2.17}$$

and then they are also subleading compared to the large-logs.

As for the power-law running corrections, their coefficients γ_a highly depend on the way of UV cutoff. Of course, if the bulk gauge group is a simple group, γ_a will be a-independent. In other cases that the bulk gauge group is not unified, one may compute γ_a in certain regularization scheme and argue that different 5D gauge couplings rapidly approach to each other (when the scale is increased) due to the power-law running governed by γ_a [17]. However, power-law running can not be considered as a calculable property of orbifold field theory since it is highly sensitive to the unknown UV physics [18, 19]. This can be easily noticed

[5] Here we consider the case with $M_* \gg k$ to see the gauge coupling renormalization proportional to $\ln(M_*/k)$. However in most of the practical applications of the model, one assumes $k \sim M_*$. The gauge coupling renormalization in such case can be obtained from our result by simply ignoring $\ln(M_*/k)$

by changing the cutoff $M_* \to c_a M_*$ for a-dependent constants c_a which are generically of order unity. This change of cutoff leads to $\gamma_a \to c_a\gamma_a$ and represents the effects of unknown threshold effects at M_*. As usual, the cutoff-scheme dependence of γ_a should be cancelled by the cutoff scheme dependence of the corresponding Wilsonian couplings $g_{5a}^2(M_*)$. It is thus not meaningful to split the power-law running part from the bare coupling $1/g_{5a}^2$ in orbifold field theory. Rather, one has to consider the cutoff-scheme independent combination

$$\kappa_a M_* \equiv \frac{1}{g_{5a}^2(M_*)} + \frac{\gamma_a}{8\pi^3} M_* \,. \tag{2.18}$$

Summarizing the above discussions, when we have the scale hierachy (2.13), the 1-loop low energy gauge couplings in AdS_5 can be written as

$$\frac{1}{g_a^2(\mu)} = \kappa_a \pi M_* R + \frac{1}{8\pi^2}\Delta_a + \mathcal{O}\left(\frac{1}{8\pi^2}\right), \tag{2.19}$$

where

$$\Delta_a = b_a''' \ln\frac{M_*}{k} + b_a'' \ln(kR) + b_a' \ln\frac{1}{M_{KK}R} + b_a \ln\frac{M_{KK}}{\mu}\,, \tag{2.20}$$

for $\mu \ll M_{KK} \approx \pi k e^{-\pi kR}$. Here the 4D momentum μ is measured in the metric frame of massless 4D gravition, and κ_a are uncalculable bare parameters associated with the bare 5D gauge couplings g_{5a}^2. The other uncalculable bare brane couplings $g_{0a}^2, g_{\pi a}^2$ are assumed to give subleading corrections of order $1/8\pi^2$. The 1-loop correction Δ_a include both the KK threshold effects and the running effects which can be unambiguously calculated within the orbifold field theory. In the next section, we will calculate Δ_a using the 4D effective SUGRA of 5D theory on AdS_5.

3. Matching with 4D effective SUGRA

In this section, we derive the 4D effective SUGRA of the 5D SYM theory on a slice of AdS_5, and calculate the low energy gauge coupling (2.16) in the framework of 4D effective SUGRA. Generic 4D SUGRA action is determined by the real Kähler potential K, the holomorphic gauge kinetic function f_a and the holomorphic superpotential P. In superspace, the action takes the form

$$\int d^4x d^4\theta \left\{-3\exp\left(-\frac{K}{3}\right)\right\} + \left[\int d^4x d^2\theta \left(\frac{1}{4} f_a W^{a\alpha} W^a_\alpha + P\right) + \mathrm{h.c}\right] \tag{3.1}$$

where the gravity multiplet fields are set to their Poincare invariant VEV. The gauge kinetic function f_a determines the 4D Wilsonian gauge coupling (g_{Wa}) and the vacuum angle (Θ_a) as

$$f_a = \frac{1}{g_{Wa}^2} + i\frac{\Theta_a}{8\pi^2}\,, \tag{3.2}$$

and the Kähler potential can be expanded in powers of gauge-charged chiral superfields:

$$K = K_0(T,T^*) + Z_\Phi(T,T^*)\Phi^* e^{-V}\Phi + \cdots\,, \tag{3.3}$$

where T, Φ, and V denote the gauge-singlet moduli superfields, gauge-charged chiral matter superfields, and vector gauge superfields, respectively.

It has been found in [20] a relation between the beta function and the anomalous dimension in 4D $N = 1$ supersymmetric gauge theory, which is exact in perturbation theory. Using this relation, one can express the low energy one-particle-irreducible (1PI) gauge couplings in terms of f_a and the Kähler metric Z_Φ of charged matter fields. This procedure can be generalized to find a relation between the moduli-dependent 1PI gauge coupling and the moduli-dependent Wilsonian couplings in 4D SUGRA [21]. For *moduli-independent* UV regulator, the super-Weyl and Kähler invariance of 4D SUGRA determines the 1PI gauge coupling as[6]

$$\frac{1}{g_a^2(\mu)} = \mathrm{Re}(f_a)+\frac{1}{8\pi^2}\left[b_a \ln\left(\frac{M_{\mathrm{Pl}}}{e^{-K_0/6}\mu}\right) - \sum_\Phi T_a(\Phi)\ln\left(\frac{Z_\Phi}{e^{K_0/3}}\right) + T_a(\mathrm{Adj})\ln\left(\frac{1}{g_a^2(\mu)}\right)\right], \tag{3.4}$$

where M_{Pl} is the 4D Planck scale of the metric $g_{\mu\nu}$ which is used to measure the external momentum $\mu^2 = -g^{\mu\nu}\partial_\mu\partial_\nu$, $T_a(\Phi) = \mathrm{Tr}([T_a(\Phi)]^2)$ is the Dynkin index of the gauge group representation Φ, and b_a is the one-loop beta function coefficient:

$$b_a = -3T_a(\mathrm{Adj}) + \sum_\Phi T_a(\Phi)\,.$$

Note that the moduli-dependence of M_{Pl} differs in different metric frame. For instance, in the 4D superconformal frame in which the action is given by (3.1), the moduli-dependence of M_{Pl} is given by $e^{-K_0/6}$, while in the 4D Einstein frame which is obtained from the superconformal frame after the Weyl scaling $g_{\mu\nu} \to e^{K_0/3} g_{\mu\nu}$, M_{Pl} is moduli-independent. When one matches the 4D SUGRA coupling (3.4) with the 1PI coupling computed in underlying orbifold field theory, one has to use the same metric frame as well as the same *moduli-independent* regulator [22]. In the one-loop approximation, the above low energy gauge couplings can be written as

$$\frac{1}{g_a^2(\mu)} = \mathrm{Re}(f_a) + \frac{1}{8\pi^2}\left[b_a \ln\left(\frac{M_{\mathrm{Pl}}}{e^{-K_0/6}\mu}\right) - \sum_\Phi T_a(\Phi)\ln\left(\frac{Z_\Phi^{(0)}}{e^{K_0/3}}\right) + T_a(\mathrm{Adj})\ln\left(\mathrm{Re}(f_a)\right)\right], \tag{3.5}$$

where $Z_\Phi^{(0)}$ is the tree level Kähler metric of Φ.

Using (3.5), one can find the 1-loop gauge coupling in 5D gauge theory on a slice of AdS_5 by computing f_a, K_0 and $Z_\Phi^{(0)}$ of the 4D effective SUGRA. As was noted in the previous section, all scale hierarchies in 5D theory are determined by the orbifold radius R, so the dominant 1-loop renormalization can be determined by computing the R-dependence of the 4D effective SUGRA. The only part of (3.5) which is undetermined by tree-level analysis is the 1-loop threshold correction to f_a. However due to the holomorphicity of f_a, in our case of 5D theory on AdS_5, the 1-loop piece of f_a is determined by the loop-induced axion coupling which can be easily computed by using the chiral anomaly structure of the theory.

[6]This relation was confirmed by an explicit computation in [21] at 1-loop order.

In order to derive the 4D effective SUGRA action of the 5D theory on AdS_5, it is convenient to rewrite the 5D actions $S_{\rm vector}$ and $S_{\rm hyper}$ in terms of the $N = 1$ superfields of unbroken SUSY [23]. This procedure involves a R and B_5-dependent field redefinition which would generate a new piece of brane action through the chiral anomaly. The Killing spinor generating the unbroken $N = 1$ global SUSY is given by (see appendix)

$$e^{-\frac{1}{2}k|y|(R-3iB_5)}\eta\,,$$

where η is a constant 4D Weyl spinor. It is then straightforward to see that the 5D vector multiplet $\{\phi^a, A^a_M, \lambda^{ia}\}$ can be decomposed into an $N = 1$ vector superfield V^a and a chiral superfield χ^a whose component fields are given by $\{A^a_\mu, \tilde{\lambda}^a\}$ and $\{\tilde{\phi}^a + iA_5, \tilde{\zeta}^a\}$, respectively, where

$$\begin{aligned}
\tilde{\lambda}^a(x,y) &= e^{-\frac{3}{2}k|y|(R+iB_5)}\lambda^a(x,y)\,,\\
\tilde{\zeta}^a(x,y) &= Re^{-\frac{3}{2}k|y|(R-iB_5)}\zeta^a(x,y)\,,\\
\tilde{\phi}^a(x,y) &= R\phi^a(x,y)\,.
\end{aligned} \tag{3.6}$$

for $\lambda^a = \frac{1}{2}(1+\gamma_5)\lambda^{1a}$ and $\zeta^a = \frac{1}{2}(1+\gamma_5)\lambda^{2a}$. The 5D hypermultiplet $\{h^i_I, \Psi_I\}$ can be similarly decomposed into two $N = 1$ chiral superfields H_I and H^c_I whose component fields are given by $\{\tilde{h}^1_I, \tilde{\psi}_I\}$ and $\{\tilde{h}^{2*}_I, \tilde{\psi}^c_I\}$, respectively, where

$$\begin{aligned}
\tilde{h}^1_I(x,y) &= e^{-\left(\frac{3}{2}-c\right)k|y|(R+iB_5)}h^1_I(x,y)\,,\\
\tilde{h}^2_I(x,y) &= e^{\left(\frac{3}{2}+c\right)k|y|(R+iB_5)}h^2_I(x,y)\,,\\
\tilde{\psi}_I(x,y) &= e^{-k|y|((2-c_I)R-ic_IB_5)}\psi_I(x,y)\,,\\
\tilde{\psi}^c_I(x,y) &= e^{-k|y|((2+c_I)R+ic_IB_5)}\psi^c_I(x,y)\,,
\end{aligned} \tag{3.7}$$

for $\psi_I = \frac{1}{2}(1+\gamma_5)\Psi_I$ and $(\psi^c_I)^* = \frac{1}{2}(1-\gamma_5)\Psi_I$.

On each brane, there can be a brane action containing $N = 1$ supersymmetric (B_5-independent) gaugino-scalar-fermion interactions $\phi^*_{\rm br}\psi_{\rm br}\lambda^a$ and/or $\phi^*_{\rm br}\psi_{\rm br}\lambda_{\rm br}$, where $\{\phi_{\rm br}, \psi_{\rm br}\}$ and $\{A^\mu_{\rm br}, \lambda_{\rm br}\}$ are the $N = 1$ matter and gauge multiplets, respectively, living only on the brane. In order to rewrite such $\mathrm{U}(1)_R$-invariant brane actions in $N = 1$ superspace, one needs also to redefine the brane fermions as follows:

$$\begin{aligned}
\tilde{\psi}_{\rm br} &= e^{-\frac{1}{2}k|y_{\rm br}|(R-3iB_5)}\psi_{\rm br}\,,\\
\tilde{\lambda}_{\rm br} &= e^{-\frac{3}{2}k|y_{\rm br}|(R+iB_5)}\lambda_{\rm br}\,,
\end{aligned} \tag{3.8}$$

where $y_{\rm br} = 0$ or π denotes the location of the brane where the field lives on. Then the $N = 1$ chiral brane superfield $Q_{\rm br}$ is given by the redefined component fields $\{\phi_{\rm br}, \tilde{\psi}_{\rm br}\}$ and the $N = 1$ vector brane superfield $V_{\rm br}$ by $\{A^\mu_{\rm br}, \tilde{\lambda}_{\rm br}\}$.

After the above field redefinitions, the bulk actions $S_{\rm vector}$ and $S_{\rm hyper}$ can be written in $N=1$ superspace [12, 23] as follows

$$
\begin{aligned}
S_{\rm bulk} = \int d^5x \Bigg[\int d^4\theta \Bigg\{ \frac{1}{2}(T+T^*)e^{-(T+T^*)k|y|}\Big(M_*^3 + M_* e^{(\frac{3}{2}-c_I)(T+T^*)k|y|} H_I^* e^{-V} H_I + \\
+ M_* e^{(\frac{3}{2}+c_I)(T+T^*)k|y|} H_I^c e^{V} H_I^{c*})\Big) + \\
+ 2\kappa_a M_* \frac{e^{-(T+T^*)k|y|}}{T+T^*}\Big(\partial_y V^a - \frac{1}{\sqrt{2}}(\chi^a + \chi^{a*})\Big)^2 \Bigg\} + \\
+ \int d^2\theta \left\{ \frac{\kappa_a M_*}{4} T W^{a\alpha} W^a_\alpha + H^c\left(\partial_y - \frac{1}{\sqrt{2}}\chi\right) H + \text{h.c.} \right\} \Bigg],
\end{aligned}
\tag{3.9}
$$

where T is the radion superfield given by

$$
T = R + iB_5 + \theta\psi_5 + \theta^2 F_T\,,
$$

for the fifth-component of the graviphoton (B_5) and $\psi_5 = \frac{1}{2}(1+\gamma_5)\Psi_5^{i=2}$. Here we included the pure radion action coming from $S_{\rm sugra}$. The brane actions can be also written in $N=1$ superspace as

$$
\begin{aligned}
S_{\rm brane} = \int_{y=0} d^4x \Bigg[\int d^4\theta Q_{UV}^* e^{-V} Q_{UV} + \int d^2\theta \left(\frac{1}{4\hat{g}_{UV}^2} W_{UV}^\alpha W_{UV\alpha} + P_{UV}(Q_{UV}, H_I) \right) \Bigg] + \\
+ \int_{y=\pi} d^4x \Bigg[\int d^4\theta e^{-k\pi(T+T^*)} Q_{IR}^* e^{-V} Q_{IR} + \\
+ \int d^2\theta \Bigg(\frac{1}{4\hat{g}_{IR}^2} W_{IR}^\alpha W_{IR\alpha} + e^{-3k\pi T} P_{IR} \times \left(Q_{IR}, e^{(\frac{3}{2}-c)\pi k T} H_I \right) \Bigg) \Bigg] + \text{h.c.}\,,
\end{aligned}
\tag{3.10}
$$

where Q_{UV} and Q_{IR} are the brane chiral superfields living on the UV ($y=0$) and IR ($y=\pi$) brane, respectively, and W_{UV} and W_{IR} are the chiral spinor superfields for the brane vector superfields V_{UV} and V_{IR}.

At classical level, the superfield action $S_{\rm bulk} + S_{\rm brane}$ describes the same theory as the component field action before the field redefinitions (3.6), (3.7) and (3.8). However at quantum level, we should include the anomaly terms induced by these field redefinitions at one-loop order. Since there is no chiral anomaly in 5D bulk, anomalies appear only at the orbifold fixed points [24, 25], which can be easily calculated to be

$$
\begin{aligned}
S_{\rm anomaly} = \int d^5x \int d^2\theta \Bigg[\Bigg\{ \frac{3}{4} T_a(\lambda^a)\left(\delta(y) + \delta(y-\pi)\right) - \frac{1}{2}\omega_I c_I T_a(\Psi_I) \times \\
\times \left(\delta(y) + \eta_I \delta(y-\pi)\right) - \frac{3}{2} T_a(\psi_{IR})\delta(y-\pi) \Bigg\} \frac{k|y|T}{16\pi^2} W^{a\alpha} W^a_\alpha + \\
+ \frac{3}{2}\left(T_{IR}(\lambda_{IR}) - T_{IR}(\psi_{IR})\right)\delta(y-\pi)\frac{k\pi T}{16\pi^2} W_{IR}^\alpha W_{IR\alpha} \Bigg].
\end{aligned}
\tag{3.11}
$$

It is now straightforward to derive the tree level 4D effective action from the above 5D actions in $N=1$ superspace. The bulk superfields T and W^a_α have (y-independent) zero modes as they have $(+,+)$ boundary condition under $y \to -y$ and $y+\pi \to -y+\pi$. On the other hand, H_I can have (y-independent) zero mode only when $(\omega_I, \eta_I) = (+,+)$, while H_I^c does only when $(\omega_I, \eta_I) = (-,-)$. By integrating over y, one easily finds the following radion Kähler potential K_0, the tree level Kähler metric $Z_{H_I}^{(0)}$, and the tree level gauge kinetic function $f_a^{(0)}$ for the massless 4D modes of T, H_I or H_I^c, and W_α^a:

$$\begin{aligned}
e^{-K_0/3} &= \left(1 - e^{-\pi k(T+T^*)}\right) \frac{M_*^3}{k} = M_{\rm Pl}^2 \,, \\
e^{-K_0/3} Z_{H_I}^{(0)} &= \frac{M_*}{(\frac{1}{2} - \omega_I c_I) k} (e^{(\frac{1}{2} - \omega_I c_I)\pi k(T+T^*)} - 1) \,, \\
f_a^{(0)} &= \kappa_a \pi M_* T \,.
\end{aligned} \tag{3.12}$$

Here we ignored the contribution to $f_a^{(0)}$ from the fixed point gauge couplings $1/g_{ia}^2$ based on the strong coupling assumption (2.17). The contribution to $Z_{H_I}^{(0)}$ from the fixed point kinetic terms of bulk fields are ignored also by the similar reasoning. The tree-level Kähler metrics and gauge kinetic functions of brane superfields are also easily obtained to be

$$\begin{aligned}
e^{-K_0/3} Z_{Q_{UV}}^{(0)} &= 1 \,, \qquad e^{-K_0/3} Z_{Q_{IR}}^{(0)} = e^{-\pi k(T+T^*)} \,, \\
f_{IR}^{(0)} &= \frac{1}{\hat{g}_{IR}^2} \,, \qquad f_{UV}^{(0)} = \frac{1}{\hat{g}_{UV}^2} \,.
\end{aligned} \tag{3.13}$$

The T-dependent one-loop threshold corrections to gauge kinetic functions can be determined by the loop-induced axion (B_5) couplings to gauge fields: $B_5 F^{a\mu\nu} \tilde{F}_{\mu\nu}^a$. There can be two sources of such axion couplings. One is $S_{\rm anomaly}$ and the other is the one-loop threshold effects of massive KK modes. A nice feature our field basis is that B_5 does not have any non-derivative coupling in $S_{\rm bulk}$ other than the Chern-Simons coupling. As a result, in our field basis, integrating out the massive KK modes does not generate any B_5-coupling to gauge fields. Then the T-dependent one-loop corrections to gauge kinetic functions are entirely given by $S_{\rm anomaly}$, yielding:

$$\begin{aligned}
\Delta f_a &= -\frac{3}{8\pi^2} \left(T_a(Q_{IR}) - \frac{1}{2} T_a({\rm Adj}) + \frac{1}{3} \eta_I c_I T_a(H_I) \right) k\pi T \,, \\
\Delta f_{IR} &= \frac{3}{8\pi^2} \left(T_a({\rm Adj}) - T_a(Q_{IR}) \right) k\pi T \,, \\
\Delta f_{UV} &= 0 \,.
\end{aligned} \tag{3.14}$$

With (3.5), (3.12), (3.13) and (3.14), the low energy bulk gauge couplings are given by

$$\begin{aligned}
\frac{1}{g_a^2(\mu, k, R)} = \kappa_a \pi M_* {\rm Re}(T) + \frac{1}{8\pi^2} \Bigg[& T_a(Q_{UV}) \ln\left(\frac{M_*}{\mu}\right) + T_a(Q_{IR}) \ln\left(\frac{M_* e^{-\frac{1}{2}\pi k(T+T^*)}}{\mu}\right) - \\
& - T_a({\rm Adj}) \left\{ 3 \ln\left(\frac{M_*}{\mu}\right) - \frac{3}{4}\pi k(T+T^*) - \ln(M_*(T+T^*)) \right\} + \\
& + \sum_{\omega_I = \eta_I} T_a(H_I) \left\{ \ln\left(\frac{k}{\mu}\right) - \ln\left(\frac{e^{(\frac{1}{2} - \omega_I c_I)\pi k(T+T^*)} - 1}{\pi(1 - 2\omega_I c_I)}\right) \right\} - \\
& - \sum_I \frac{1}{2} T_a(H_I) \omega_I \eta_I c_I \pi k(T+T^*) \Bigg] \,,
\end{aligned} \tag{3.15}$$

where $\mu^2 = -g^{\mu\nu}\partial_\mu\partial_\nu$ is the external 4D momentum below the KK threshold scale. Here $\sum_I$ denotes the summation over all hypermultiplets, while $\sum_{\omega_I=\eta_I}$ denotes the summation over the hypermultiplets having a zero mode. The brane gauge couplings at low energies can be similarly obtained as

$$\begin{aligned}
\frac{1}{g^2_{UV}(\mu,k,R)} &= \frac{1}{\hat{g}^2_{UV}} + \frac{b_{UV}}{16\pi^2}\ln\left(\frac{M_*^2}{\mu^2}\right), \\
\frac{1}{g^2_{IR}(\mu,k,R)} &= \frac{1}{\hat{g}^2_{IR}} + \frac{b_{IR}}{16\pi^2}\ln\left(\frac{e^{-\pi k(T+T^*)}M_*^2}{\mu^2}\right),
\end{aligned} \tag{3.16}$$

where $b_A = -3T_A(\text{Adj}) + \sum_\Phi T_A(\Phi)$ (A = UV or IR) are the one-loop beta function coefficients for the couplings of gauge fields living only on the branes.

The above expression of low energy brane couplings is what one would expect based on the notion of position-dependent effective cutoff [14]. The bare brane couplings $\hat{g}^2_A$ (A = UV or IR) corresponds to the Wilsonian couplings at the cutoff scale M_* in the 5D metric frame of G_{MN}. However in the metric frame of $g_{\mu\nu}$ in which $\mu^2 = -g^{\mu\nu}\partial_\mu\partial_\nu$ is measured (see eq. (2.3)), $\hat{g}^2_{IR}$ can be identified as the Wilsonian coupling at the effective cutoff scale $e^{-\pi kR}M_*$, while $\hat{g}^2_{UV}$ is still the Wilsonian coupling at M_*.

As for the low energy bulk gauge couplings, our result (3.15) shows that the (calculable) quantum corrections are generically of order $\frac{1}{8\pi^2}\ln(M_*^2/\mu^2) \sim \frac{1}{8\pi^2}\ln(M_{\rm Pl}^2/M_{\rm weak}^2)$. It reproduces correctly the known 1-loop gauge coupling in flat 5D orbifold in the limit $kR \to 0$. In this flat limit, (3.15) is reduced to

$$\frac{1}{g_a^2(\mu)} = \kappa_a \pi M_* \,\text{Re}(T) + \frac{1}{8\pi^2}\left[b_a \ln\left(\frac{1}{\mu R}\right) + b_a'' \ln\left(M_* R\right)\right] + \cdots \tag{3.17}$$

where the ellipsis stands for the subleading pieces of $\mathcal{O}(1/8\pi^2)$, and

$$b_a'' = -2T_a(\text{Adj}) + \sum T_a\left(Q_{UV}\right) + \sum T_a\left(Q_{IR}\right).$$

This value of b_a'' agrees with what one would obtain in the explicit KK loop computation [19]. In fact, (3.15) can be obtained also through a totally independent (but more general) 5D calculation using dimensional regularization [22], providing a nontrivial check of our result.

The result (3.15) is valid for arbitrary value of k and μ as long as $k \lesssim M_*$ and $\mu \lesssim M_{KK}$. When $\pi k(T+T^*) \gg 1$ so that we have the scale hierarchy (2.13), one can rewrite (3.15) in terms of the logarithms of four distinctive mass scales $M_{KK} \approx \pi k e^{-\pi k(T+T^*)/2}, 1/R$, k and M_*. In this regard, for the hypermultiplets with $\omega_I = \eta_I$ having a massless mode, it is convenient to consider the following three different classes:

$$\begin{aligned}
\text{Class 1:} \quad & \frac{1}{2} - \omega_I c_I \gg \frac{1}{\pi k(T+T^*)}, \\
\text{Class 2:} \quad & \left|\frac{1}{2} - \omega_I c_I\right| \ll \frac{1}{\pi k(T+T^*)}, \\
\text{Class 3:} \quad & \frac{1}{2} - \omega_I c_I \ll \frac{-1}{\pi k(T+T^*)}.
\end{aligned} \tag{3.18}$$

Then (3.15) can be written as

$$\frac{1}{g_a^2(\mu)} = \kappa_a \pi M_* \,\mathrm{Re}(T) + \frac{b_a}{8\pi^2}\ln\left(\frac{M_{KK}}{\mu}\right) + \frac{b'_a}{8\pi^2}\ln\left(\frac{1}{M_{KK}R}\right) + \\ + \frac{b''_a}{8\pi^2}\ln(kR) + \frac{b'''_a}{8\pi^2}\ln\left(\frac{M_*}{k}\right) + \cdots\,, \tag{3.19}$$

where

$$\begin{aligned} b_a &= -3T_a(\mathrm{Adj}) + \sum T_a(Q_{IR}) + \sum T_a(Q_{UV}) + \sum_{\omega_I=\eta_I} T_a(H_I)\,, \\ b'_a &= -\frac{3}{2}T_a(\mathrm{Adj}) + \sum T_a(Q_{UV}) + \Bigg[-\sum_I \omega_I\eta_I c_I T_a(H_I) + \sum_{\omega_I=\eta_I} T_a(H_I) - \\ &\qquad - \sum_{\text{class 1}} (1-2\omega_I c_I) T_a(H_I)\Bigg]\,, \\ b''_a &= -\frac{1}{2}T_a(\mathrm{Adj}) + \sum T_a(Q_{UV}) + \Bigg[-\sum_I \omega_I\eta_I c_I T_a(H_I) + \sum_{\omega_I=\eta_I} T_a(H_I) - \\ &\qquad - \sum_{\text{class 1}} (1-2\omega_I c_I) T_a(H_I) - \sum_{\text{class 2}} T_a(H_I)\Bigg]\,, \\ b'''_a &= -2T_a(\mathrm{Adj}) + \sum T_a(Q_{UV}) + \sum T_a(Q_{IR})\,, \end{aligned} \tag{3.20}$$

and the ellipsis stands for the subleading pieces of $\mathcal{O}(1/8\pi^2)$. Obviously b_a represents the conventional coupling running at scales between μ and M_{KK}. On the other hand, other coefficients contain the KK threshold effects, so their logarithms can not be interpreted as a coupling running.

4. Conclusion

In this paper, we have examined the gauge coupling renormalization in orbifold field theory in the context of 4D effective SUGRA. The 4D effective SUGRA is a convenient framework to study the gauge coupling renormalization since the 1-loop gauge couplings can be determined by the 1-loop induced axion couplings and the tree level properties of 4D effective SUGRA which are much easier to be computed. To be explicit, we take an example of the 5D SUGRA-coupled SYM theory on a slice of AdS_5 with four well-seperated mass scales, the KK threshold scale, the orbifold length, the AdS curvature, and the cutoff scale. In this case, the relevant axion couplings are those of the graviphoton which can be determined simply by the chiral anomaly structure of the model. The calculable piece of gauge coupling renormalization in AdS_5 is logarithmic, which is generically of order $\frac{1}{8\pi^2}\ln(M_{\rm Pl}^2/M_{\rm weak}^2)$, so is numerically of order unity. We have calculated such logarithmic corrections in generic 5D SYM models defined on a slice of AdS_5 preserving $N=1$ SUSY.

Note added: After this paper is completed, there have appeared several papers discussing the gauge coupling renormalization in AdS_5 [27, 28, 29, 30, 31, 22, 32, 33].

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Acknowledgments

HK thanks the particle theory group of University of Washington, Seattle for their warm hospitality and thanks D. B. Kaplan, A. Katz and A. Nelson for discussions. This work is supported in part by the BK21 program of Ministry of Education, KRF Grant No. 2000-015-DP0080, the KOSEF Sundo Grant, and the Center for High Energy Physics(CHEP), Kyungpook National University.

A. 5D SUSY transformation and gauged $\mathrm{U}(1)_R$

In this appendix, we briefly discuss how the $\mathrm{U}(1)_R$-gauging (2.11) is related with the AdS curvature and the mass parameter of the hypermultiplets through the 5D SUGRA transformation. Let us first set up the notation for 5D spinor. Generic spinor field in 5D SUGRA can be represented by symplectic-Majorana spinor satisfying

$$\bar{\Psi}^i = \Psi_i^\dagger \gamma^0 = (\Psi^i)^T C \,.$$

where

$$C^T = -C\,, \quad (\gamma^A)^T = C\gamma^A C^{-1}\,,$$

and the $\mathrm{SU}(2)_R$ doublet index $i = 1, 2$ is raised or lowered by $\varepsilon^{ij} = \varepsilon_{ij} = (i\sigma_2)_{ij}$:

$$\Psi^i = \varepsilon^{ij}\Psi_j\,, \qquad \Psi_i = \Psi^j \varepsilon_{ji}\,.$$

A symplectic-Majorana spinor contains two *independent* left-handed 4D Weyl spinors ψ and ψ^c:

$$\Psi^{i=1} = \begin{pmatrix} \psi \\ \bar{\psi}^c \end{pmatrix}, \qquad \Psi^{i=2} = \begin{pmatrix} \psi^c \\ -\bar{\psi} \end{pmatrix},$$

so $\Psi^{i=1}$ or $\Psi^{i=2}$ can be regarded as a Dirac spinior.

The 5D SUGRA action of general vector multiplets and hypermultiplets and their SUGRA transformations are given in [26]. In our notation, the SUGRA transformation of the gravity multiplet $\{e_M^A, \Psi_M^i, B_M\}$ is given by

$$\begin{aligned} \delta_\xi e_M^A &= \frac{1}{2}\bar{\xi}^i \gamma^A \Psi_{iM}\,, \qquad \delta_\xi B_M = \frac{i}{2}\overline{\Psi}^i_M \xi_i\,, \\ \delta_\xi \Psi_M^i &= D_M \xi^i + \frac{1}{2}k\epsilon(y)\left(\sigma^3\right)^i{}_j\, e_M^A \gamma_A \xi^j + \frac{i}{8} e_M^A \left(\gamma_{ABC}\xi^i - 4\eta_{AB}\gamma_C \xi^i\right) C^{BC} + \cdots\,, \end{aligned} \tag{A.1}$$

where $C_{MN} = \partial_M B_N - \partial_N B_M$ and the ellipsis denotes the piece which is bilinear in hyperino fields. Comparing $D_M\xi^i$ in $\delta_\xi \Psi_M^i$ with the next term, one easily finds

$$D_M \xi^i = \nabla_M \xi^i - i\frac{3}{2}k\epsilon(y)(\sigma^3)^i{}_j B_M \xi^i\,,$$

and this determines the $\mathrm{U}(1)_R$-gauging for the fields carrying $\mathrm{SU}(2)_R$ index i, e.g

$$D_M \lambda^{ia} = \nabla_M \lambda^{ia} - i\frac{3}{2}k\epsilon(y)(\sigma^3)^i{}_j B_M \lambda^{ja}\,.$$

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The SUGRA transformation of hypermultiplet $\{h^i_I, \Psi_I\}$ is given by [26]

$$\begin{aligned}\delta_\xi h^i_I &= -i\bar{\xi}^i\Psi_I\,,\\ \delta_\xi\Psi_I &= -\frac{i}{2}\left[\gamma^A\xi^i D_A h^j_I\varepsilon_{ji} + k\epsilon(y)\xi^i\left(\frac{3}{2}(\sigma_3)^j{}_k - c_I\delta^j_k\right)h^k_I\varepsilon_{ji}\right].\end{aligned}\tag{A.2}$$

Again comparing the first term in $\delta_\xi\Psi_I$ with the second, one easily finds

$$D_M h^i_I = \partial_M - ik\epsilon(y)\left(\frac{3}{2}(\sigma_3)^i{}_j - c_I\delta^i_j\right)k\epsilon(y)B_M h^j_I\,,$$

and so

$$D_M\Psi_I = \partial_M + ic_I k\epsilon(y)B_M\Psi_I\,.$$

Let us now make sure that the above $\mathrm{U}(1)_R$ covariant derivatives correctly lead to the mass parameters in S_{hyper} of (2.6). For the SUGRA transformation (A.1), the Killing spinor equation is given by

$$D_M\xi^i + \frac{1}{2}k\epsilon(y)(\sigma^3)^i{}_j e^A_M\gamma_A\xi^j = 0\,,$$

which has the solution

$$\xi^{i=1} = \begin{pmatrix}\xi\\0\end{pmatrix},\qquad \xi^{i=2} = \begin{pmatrix}0\\\bar{\xi}\end{pmatrix},$$

where

$$\xi(y) = e^{-\frac{1}{2}k|y|(R-3iB_5)}\eta\tag{A.3}$$

for a constant 4D Weyl spinor η generating the unbroken $N=1$ global SUSY.

From (A.3) and (A.2), one can see that $\left\{\tilde{h}^1_I, \tilde{\psi}_I\right\}$ and $\left\{(\tilde{h}^2_I)^*, \tilde{\psi}^c_I\right\}$ form $N=1$ chiral superfields H_I and H^c_I under the global SUSY generated by η, where $\tilde{h}^i_I$, $\tilde{\psi}_I$ and $\tilde{\psi}^c_I$ are defined in (3.7). One can also find from (A.3) and (A.2) that the F-components of these chiral superfields are given by

$$\begin{aligned}F_{H_I} &= \frac{e^{-(1-2c_I)kR|y|}}{R}\partial_5 h^{c*}_I\,,\\ F_{H^c_I} &= -\frac{e^{-(1+2c_I)kR|y|}}{R}\partial_5 h^*_I\,.\end{aligned}\tag{A.4}$$

Using the kinetic terms, it is also straightforward to find that the D-term action of hypermultiplets is given by [23, 12]

$$\int d^5x\int d^4\theta\frac{1}{2}(T+T^*)\left[e^{\left(\frac{1}{2}-c_I\right)k|y|(T+T^*)}H_I H^*_I + e^{\left(\frac{1}{2}+c_I\right)k|y|(T+T^*)}H^c_I H^{c*}_I\right].$$

In order for the F-components of H_I and H^c_I to be given as (A.4), the F-term action should be given by

$$\int d^5x\left[\int d^2\theta H^c_I\partial_5 H_I + \text{h.c.}\right].$$

It is then straightforward to see that the above D-term and F-term actions give the correct hypermultiplet masses:

$$\begin{aligned} M(\Psi_I) &= c_I k \epsilon(y)\,, \\ M^2(h_I^{i=1}) &= \left(c_I^2 + c_I - \frac{15}{4}\right) k^2 + (3 - 2c_I) k \left(\delta(y) - \delta(y-\pi)\right), \\ M^2(h_I^{i=2}) &= \left(c_I^2 - c_I - \frac{15}{4}\right) k^2 + (3 + 2c_I) k \left(\delta(y) - \delta(y-\pi)\right). \end{aligned}$$

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